

Math 120A: Final Exam Study Sheet

Here is some general information about the final.

- The final is Tuesday, December 16th, 3-6 p.m., in Geology 3656.
- I will have office hours Monday, December 15th, 2-3 p.m. and Tuesday, December 16th, 11 a.m.-12 p.m.
- The final exam will have ten questions, all of which will be variations on the “state a major theorem or definition, then prove, apply, or compute something related to it” format familiar from the first two midterms.
- Five of the questions on the final will involve the material since the second midterm (to wit, the first and second fundamental forms). The remaining five questions will be on previous material, including one question on the inverse function theorem.

There are three lectures of the course that are not covered by the homeworks. Here are a few good practice problems on those lectures: 8.1.4, 8.1.7, 8.1.9, 8.1.13, 8.2.2, 8.2.3, 8.2.7, 8.2.9. A version of one of these will appear on the final.

Here is a summary of the topics we have covered and the theorems we have proved.

- Curves
 - Definitions of curves; differences between level and parametrized curves.
 - Regular curves; unit speed curves; reparametrization preserves regularity.
 - Definition of arclength; reparametrization preserves arclength.
 - Closed curves vs. self-intersections, periods of closed curves.
 - Reading: Pressley 1.1-5.
- Curvature and Torsion
 - Definition and formulas for curvature and torsion.
 - Construction of signed curvature in the plane.
 - Relationship of the turning angle to signed curvature.
 - The Frenet-Serret frame and equations.
 - Direct isometries of \mathbb{R}^n .
 - Torsion and planar curves.
 - Signed curvature determines the plane curve; (nonzero) curvature and torsion determine the space curve.
 - Reading: Pressley 2.1-3
- Global Curve Properties
 - Definitions of simple closed curves and convexity.
 - Definition of area and the isoperimetric inequality.
 - Definition of a vertex and the four vertex theorem.

- Reading: Pressley 3.1-3.
- Surfaces and Smooth Functions
 - Definitions of regular/allowable surface patches, atlases, and surfaces.
 - Abstract definition of the tangent plane; tangent plane in coordinates.
 - Smooth maps between surfaces and the construction of the derivative map; matrix of the derivative map with respect to coordinates from a surface patch.
 - Diffeomorphisms and local diffeomorphisms; relationship to the derivative map.
 - Oriented and orientable surfaces.
 - Reading: Pressley 4.1-5.
- Examples of Surfaces
 - Level surfaces and the gradient vector.
 - Classification of the quadrics.
 - Formulas and regularity of ruled surfaces and surfaces of revolution (with the sphere and cylinder as noteworthy examples).
 - Statement of the inverse function theorem; be able to construct simple applications on the same principles of the proofs in class/homework.
 - Reading: Pressley 5.1-3, 5.6.
- Distance and the First Fundamental Form
 - Definition and motivation for the first fundamental form, abstractly and in coordinates.
 - Effect of reparametrization on the first fundamental form.
 - Definition and purpose of the pullback form.
 - Definitions of angle and area (of a small region).
 - Local isometries, conformal maps, and equiareal maps: abstract definition and corresponding conditions on the first fundamental form.
 - Great circles and spherical triangles
 - Reading: Pressley 6.1-4 (not tangent developables).
- Curvature and the Second Fundamental Form
 - Motivation and coordinate description for the first fundamental form.
 - Definitions of the Gauss map and the Weingarten map.
 - Relationship between the first and second fundamental forms.
 - Normal and geodesic curvature of a curve in a surface.
 - Normal curvature and the second fundamental form.
 - Reading: Pressley 7.1-3
- Numerical Invariants and the Second Fundamental Form Continued

- Matrix representation of the Weingarten map; Gaussian and mean curvatures of a surface.
- Principal curvatures of a surface; orthogonal bases for the tangent plane.
- Relationship between the principal curvatures and H and K .
- Relationship between the principal curvatures and curves on the surface.
- Reading: Pressley 8.1-2 [We might say a few words about 8.3 at the close of Friday, but this will not be part of the material for the final.]